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What is the Impact on Exploratory Factor Analysis Results of a Polychoric Correlation Matrix from LISREL/PRELIS and EQS When Some Respondents Are Not Able to Follow the Rating Scale?

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Abstract

This paper describes a simulation study that investigates the effect that misresponding to LIKERT or rating scales has on sample statistics from exploratory factor analyses conducted in LISREL and EQS. Specifically, misresponding was operationalized as a collapsing of the upper two scale points of a given LIKERT scale to reflect that respondents cannot discriminate between these scale points. Three factors were manipulated in the design for a total of 84 conditions, (1) percent of respondents misresponding (10%, 20%, 30%), (2) number of items that they misrespond to (1, 2, 3, 10), and (3) number of scale points (3, 4, 5, 6, 7, 8, 9). For each condition, 100,000 observations from symmetric distributions were generated. In the first part, the polychoric correlation matrices provided by PRELIS/LISREL and EQS for extreme conditions were compared and found to be of negligible difference. In the second part, using LISREL, it was found that for less than five scale points the effect of misresponding on items had the strongest impact on the sample statistic values but that such effects were of negligible magnitude as the number increased to 5 and above.

What is the impact on exploratory factor analysis results of a polychoric correlation matrix from LISREL/PRELIS and EQS when some respondents are not able to follow the rating scale?

LIKERT or rating scale data are common in the social and psychological sciences as many measurement instruments ask respondents to provide ratings regarding self-perception, attitudes, or sensations. These scales represent discretizations of an underlying continuum and the number of scale points represents a compromise between the technical attempt to approximate this continuum by providing a sufficiently large number of scale points and the practical requirement that the respondents should be able to discriminate between the number of scale points with respect to the underlying attribute an item is intended to measure. Hence, from a purely mathematical scaling perspective, the simple onus of "the more scale points the better" is certainly true; yet, from a psychometric perspective that is also concerned with the reliability and validity of the measurements taken, the issue is more complex and a "moderate" number of scale points is often recommended. In fact, even though LIKERT scales can consist of as little as two scale points, most commonly employed LIKERT scales have between 4 and 7 scale points.

When polytomously scored data from LIKERT scales are modeled, many software programs are based on an underlying assumption that the response variables are measured on an interval or ratio scale and that the joint distribution of the scores on all items is multivariate normal (West, Finch, & Curran, 1995). Unfortunately, this strict assumption may be rarely met in practice (Micceri, 1989). Deviations from this assumption can take on a number of forms such as multivariate continuous distributions that are nonnormal, which includes general elliptical distributions that are skewed and display varying degrees of kurtosis, coarsely categorized variables, of which LIKERT scale data are an example, and mixtures of the two. Hence, it is of interest to researchers in psychometric theory, particularly in the field of robust estimation, to investigate the degree to which descriptive and inferential results from model calibrations remain invariant or at least “comparable” under varying degrees of violations from these assumptions. Research has shown that there is often little effect on the bias of parameter estimates but that their standard errors are severely affected impacting all inferential results (e.g., Byrne, 1995). Even though there is no simple answer to this question, because the answer depends heavily on the complexity of the psychometric model (e.g., a Rasch model in IRT is certainly less complex than a fully developed structural equation models with multiple latent variables and complex correlation structures). Nevertheless, it is certainly a tenable conclusion that, as one would expect, the comparability of model statistics and inferential conclusions is most compromised as the number of scale points decrease to “small” numbers such as about 2 to 4 and the degree of deviation from multivariate normality become more severe. It should be noted that estimation methods such as bootstrapping, asymptotic distribution free estimation (Browne, 1984), or the general continuous categorical variable methodology (Muthén, 1984) have been developed but their requirements on sample sizes and additional theoretical assumptions make their practical implementation for smaller studies prohibitive (Bentler & Dudgeon, 1996; Dolan, 1994; West et al., 1995).

The focus of this paper is on modeling data that are generated from polytomously scored items using a covariance modeling approach (Jöreskog, 1969), which is most commonly known under the label structural equation modeling (Muthén, 2002). Within such a framework, the data to be modeled are the covariance or correlation matrix of the observed variables and for the purpose of this study we will limit ourselves to an exploratory factor analysis model based on the correlation matrix of the data. If the variables were measured on a continuous scale, Pearson product-moment correlation coefficients would be the appropriate estimators of the population correlations. However, for completely polytomous data a common alternative is to use polychoric correlation matrices and for mixed continuous and polytomous data a common alternative is to use polyserial correlation matrices. For this paper, we will focus on polytomous data only and hence we will base our work on estimation routines that employ polychoric correlation matrices.

For a polychoric correlation matrix, an underlying continuum for the polytomous scores is assumed and the observed responses are considered manifestations of respondents exceeding a certain number of latent thresholds on that underlying continuum. Conceptually, the idea is to estimate the latent thresholds and model the observed cross-classification of response categories via the underlying latent continuous variables. Formally, for item *j* with response categories *c* = 0, 1, 2, …,*C*-1, define the latent variable y* such that

$$y_j = c \quad \text{if} \quad \tau_c < y_j^* < \tau_{c+1},$$

where τ_c, τ_{c+1} are the latent thresholds on the underlying latent continuum, which are typically spaced at non-equal intervals and satisfy the constraint $-\infty = \tau_0 < \tau_1 < \cdots < \tau_{C-1} < \tau_C = \infty$.

It is worth mentioning at this point that the latent distribution does not necessarily have to be normally distributed, although it commonly is due to its well understood nature and beneficial mathematical properties, and that one should be willing to believe that this model with an underlying latent dimension is actually realistic for the data at hand. As West et al. (1995) state clearly for binary items,

> "[…] this approach will be theoretically reasonable only in some cases. For example, for many attitude items, the researchers will be more interested in the relationships among the normally distributed, continuous underlying latent variables than in the simple relationships

> between the observed "agree" versus "disagree" responses on the items. For other continuously distributed variables such as current drug use ("yes" vs. "no"), it is difficult to conceive of a *normally distributed* underlying latent variable. Finally, some variables such as gender are inherently categorical, so no continuous underlying variable could exist" (p. 69).

For the purpose of this research, which consists of a simulation study, we will hence assume that an underlying latent normally distributed continuum is reasonable.

As discussed above, the number of scale points has an impact on how well respondents are able to differentiate between adjacent scale points; thus, research is necessary that investigates the robustness properties of statistical estimators and inferential decisions under varying degrees of deviation from this ideal. We will define a situation in which respondents cannot properly distinguish between adjacent scale points as *misresponding* and focus our attention in this paper on a failure to discriminate between the upper two scale points on a LIKERT scale of given length. Effectively, this amounts to collapsing the upper two scale points into one and reducing the scale to a scale with one less scale point. While previous research has investigated the sensitivity of some statistical estimators to such misresponse processes (e.g., Brown, 1991), this study adds a new dimension to the investigation by manipulating, via simulation, the number of respondents out of the total set that misrespond in such a way.

Specifically, this study will consider an item set with 10 items, for which a one-factor model holds well in the continuous case. The design consists of three factors, (1) percent of respondents who misrespond (10%, 20%, 30%), (2) number of items they misrespond to (1, 2, 3, 10), and (3) number of scale points of items (3, 4, 5, 6, 7, 8, 9), leading to $3 \times 4 \times 7 = 84$ different scenarios or cells in the design. The distribution of the item responses for each item is taken to be symmetrical for the entire design. For each cell the design employs pseudo-population level data, which effectively means that a data set with 100,000 responses to the 10 items in each case is calibrated without any replication and estimation of Monte Carlo error. While this precludes formal effect tests of the design factors due to a lack of replication in each cell, the large sample sizes provides sufficient information to investigate effects descriptively at the pseudo-population level. Furthermore, due to the analyses conducted in this study, the simulation time was significantly reduced.

Two response variables were considered in this study, (1) the ratio of the first to second eigenvalue (REV) of the polychoric correlation matrices from a principal components eigenvalue decomposition and (2) the root mean-square error of approximation (RMSEA) from an exploratory factor analysis. Two software programs were used to perform the analyses in this study, (1) PRELIS 2 in LISREL version 8.50 (Jöreskog and Sörbom, 2001) and EQS version 5.7b (Bentler, 1989). Compared to PRELIS/LISREL, the polychoric correlations in EQS are derived simultaneously, rather than one at a time (Bentler, 1995).

We originally intended to choose statistics common to both estimation programs and to compare their overall performance, but an inspection of the polychoric correlation matrices that both programs estimate made it clear that little practical differences between the two programs could be expected. Since the estimation time with EQS exceeded that in LISREL by almost 25 minutes for the items with nine scale points on a Pentium III processor with 128 MB Ram, we decided to conduct the following two separate studies instead.

Study 1 – Comparison of PRELIS/LISREL and EQS Polychoric Correlation Matrices

If any striking differences in the Maximum Likelihood exploratory factor analysis results are extant, they are probably due to the computation of the polychoric correlation matrices in PRELIS/LISREL and EQS. While some literature comparing LISREL and EQS exists (e.g., Brown, 1986; Byrne, 1995; Waller, 1993) no detailed literature that compares the estimation process in both programs exist. The first study thus compared the estimated polychoric correlation matrices for the "corners" of the three-dimensional design surface, which represent the least and most extreme cases in every combination. In addition, the scenario with no misrepsonding for three and nine scale-point items was compared. Table 1 shows the polychoric correlation matrix for the case of most misresponding and the highest computational demands where 30 percent of the respondents misrespond to all 10 items each of which having 9 scale points; for the remaining matrices, including cases of no misresponding and continuous scales, see Tables A1-A10.

Table 1 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (30% Misresponding, All 10 Items, 9 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1					-.001				
2	.546	1								
3	.541	.455	1							
4	.514	.439	.430	1					-.001	
5	.507	.428	.421	.404	1					
6	.457	.385	.378	.360	.361	1				
7	.451	.379	.378	.363	.348	.317	1			
8	.373	.311	.310	.293	.287	.265	.260	1		
9	.352	.297	.295	.280	.274	.251	.245	.204	1	
10	.340	.292	.286	.279	.270	.242	.237	.197	.186	1

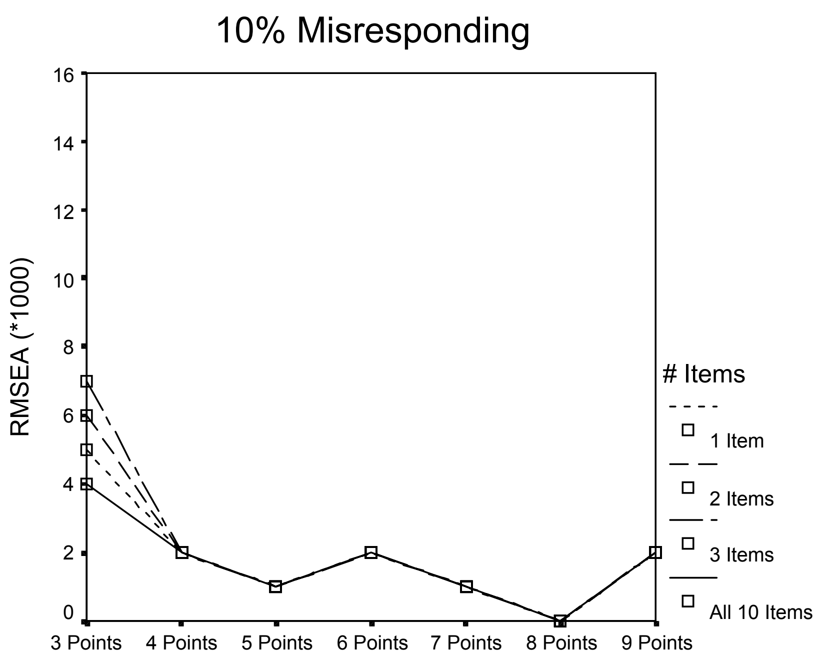
Notes: The lower off-diagonal contains the PRELIS/LISREL estimates of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between PRELIS/LISREL and EQS correlations where Δ_j = PRELIS/LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

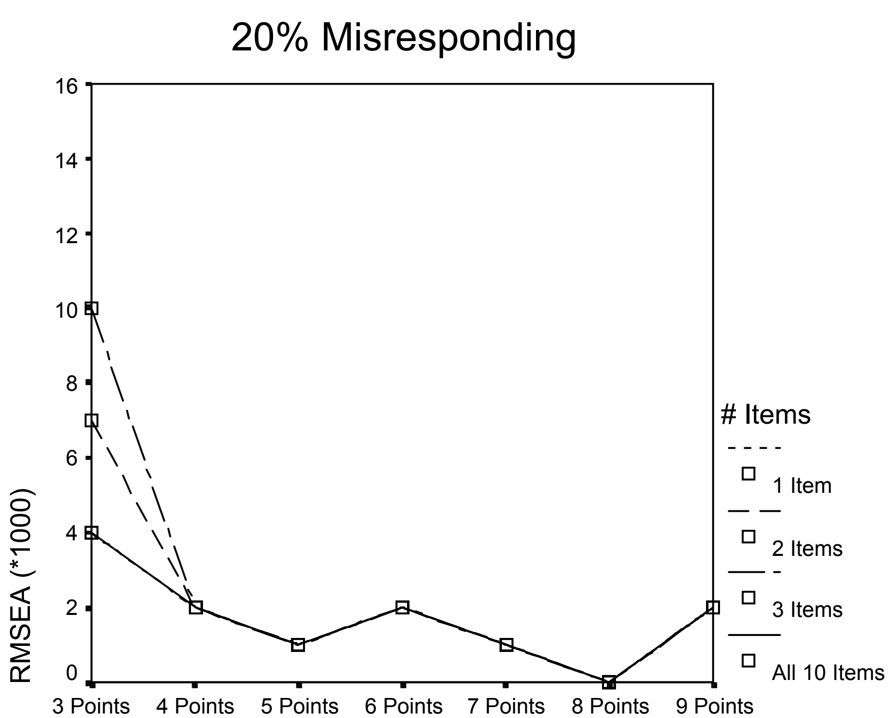
All matrices show that the numerical differences are completely negligible from any practical perspective, which speaks well for using either PRELIS/LISREL or EQS to compute such matrices for input either directly into their estimation routines or estimation routines in alternative programs. Since the computation time for PRELIS/LISREL was much faster than for EQS, all subsequent analyses in study 2 were performed using PRELIS/LISREL.

Study 2 – Exploratory Factor Analysis: RMSEA and REV Differences for Misresponding

The RMSEA and the REV were computed for each of the 84 cells described above along with the 7 baseline conditions of no misresponding for the different scale points and the 1 baseline condition of continuous response scale yielding a total of 92 calibrations.

Since the design consists of three crossed plots, either a three-dimensional plot of the results or separate two-dimensional plots can be employed; the latter were used for presentational clarity. Figure 1 thus show the RMSEA values for each of the three misresponding cases (10%, 20%, and 30%) and plot the RMSEA, on the vertical axis, as a function of the number of scale points, on the horizontal axis, and the number of items that examinees misrespond to, on separate lines. Note that the RMSEA was multiplied by 1000 to make the display easier to read.


10% Misresponding
RMSEA (*1000)
0
2
4
6
8
10
12
14
16
3 Points
4 Points
5 Points
6 Points
7 Points
8 Points
9 Points
Items
1 Item
2 Items
3 Items
All 10 Items


20% Misresponding
RMSEA (*1000)
0
2
4
6
8
10
12
14
16
3 Points
4 Points
5 Points
6 Points
7 Points
8 Points
9 Points
Items
1 Item
2 Items
3 Items
All 10 Items

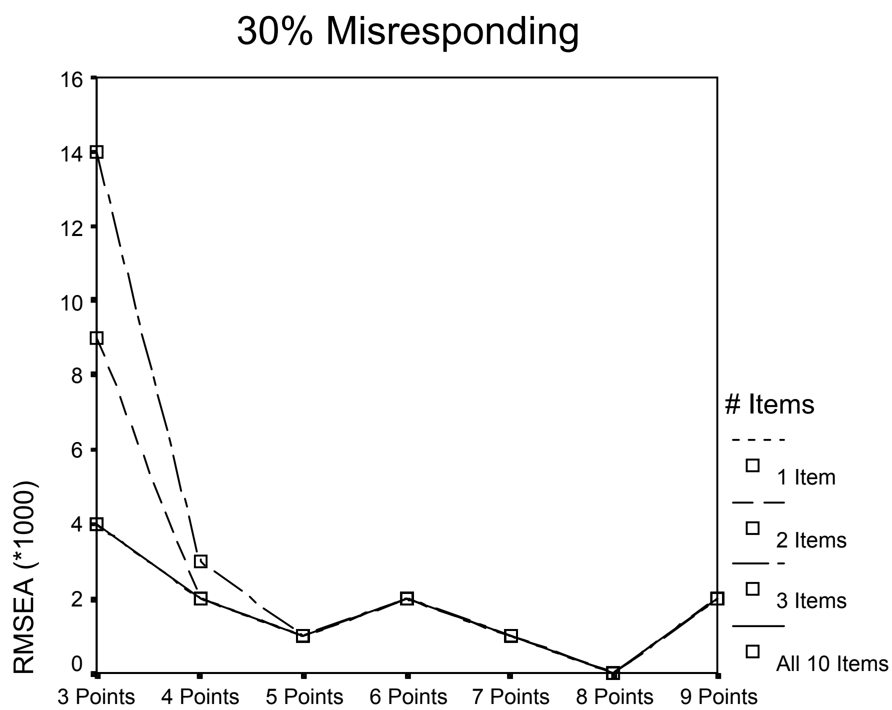
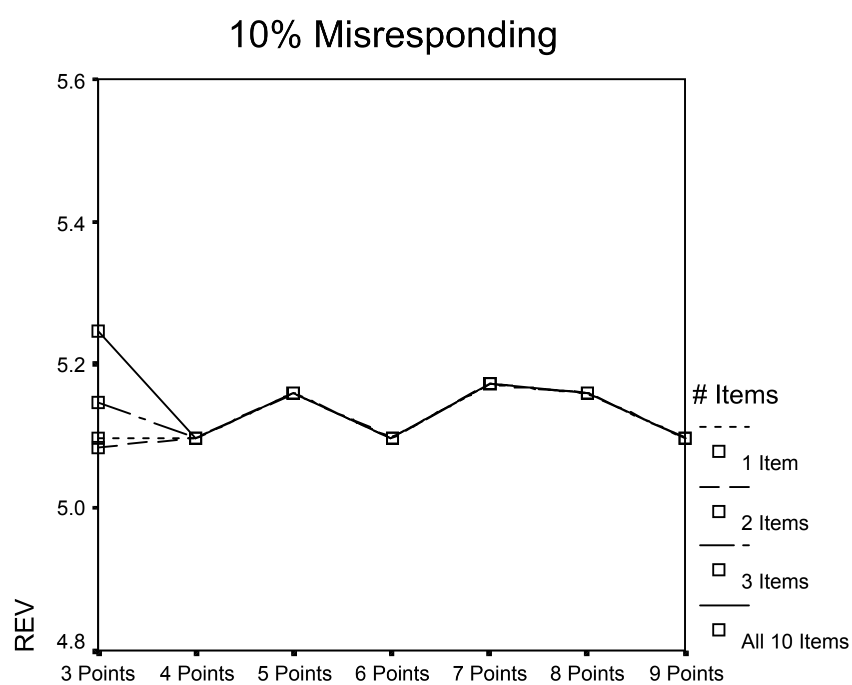
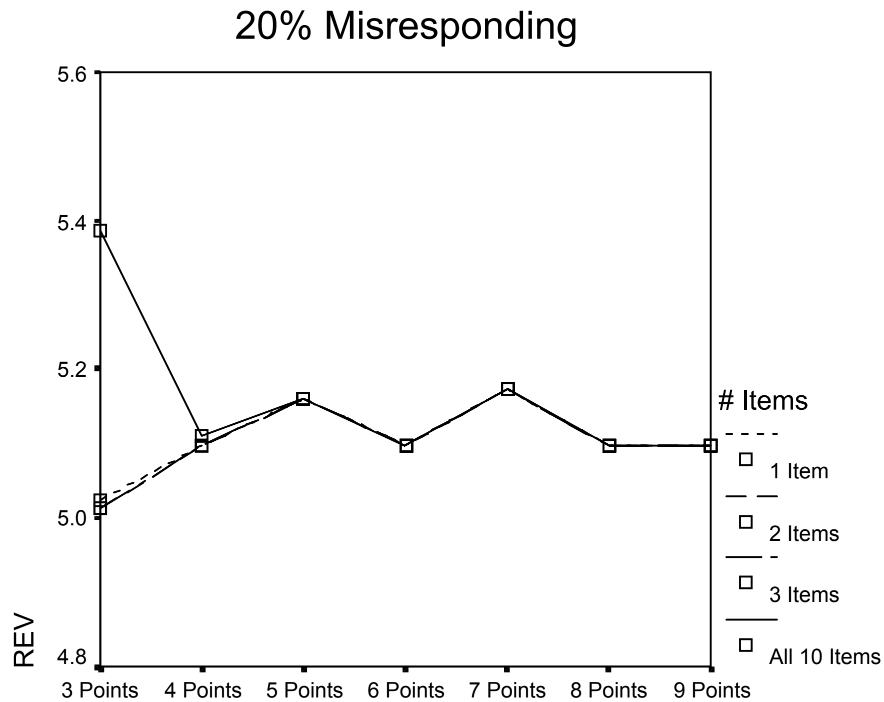


Figure 1 – RMSEA(*1000) Across Simulation Condition

These graphs show some expected and some unexpected trends. First, as expected, the variation in RMSEA values is most strongly visible when 30% of the examinees misrespond to items with either 3 or 4 scale points (resulting in items with 2 and 3 new scale points respectively) followed by less variation for the case of 20% misrespondents for 3 scale points and even less variation for 10% misrespondents and 3 scale points. Moreover, the variation also shows that the absolute value of the RMSEA can be highest for 30% misrespondents where in some cases the RMSEA would indicate poor or at best moderate fit. For example, when 30% of the examinees misrespond to 3 items, the RMSEA(*1000) = 14, which corresponds to RMSEA = .014, a poor fit. The plots also lend support to the intuitive expectation that any collapsing of scale categories due to misresponding has its strongest effect on a sample statistic such as the RMSEA for items with the least scale points, because the most variation was observed for items with 3 scale points and perhaps 4 scale points. Even though the RMSEA values fluctuated somewhat under the remaining conditions, they ranged between a value close to 0 and .002 indicating proper fit of the model, which compares favorable with the case of no misresponding for 3 and 9 scale points as well as the case of continuous data, for which RMSEA is .004, .002, and .001 respectively. The one pattern that was unexpected, however, was that the RMSEA for the 3 scale point condition did not indicate the strongest misfit for the case where all the given number of examinees misresponded to all 10 items. Instead, it was observed that the strongest misfit was indicated for 3 items that were misresponded to followed by 2 items and then 1 and all10 items.

Overall, this analysis shows that the strongest indication of inappropriate misfit is indicated for cases where a large number of respondents misresponds to some items correctly yet to others incorrectly creating a strong mixture of response processes from that perspective. Figure 2 shows the results for the REV where an identical graphical layout was used.





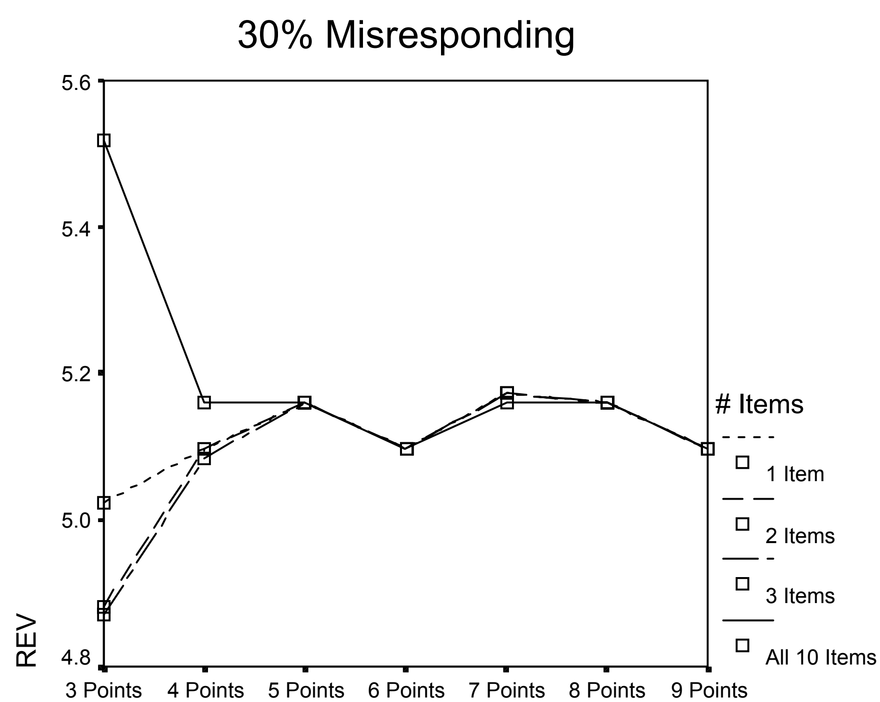


Figure 2 – REV Across Simulation Condition

These plots lend further credence to the observations that have been made above showing again that the largest variation is observable for the smallest number of scale points (i.e., 3 and 4) and the largest number of examinees misresponding. Furthermore, there is again no relative ordering of the REV that is consistent across the three conditions of misresponding in the extremes and some similar but small fluctuations of the REV across conditions are again observed for five to nine scale points.

To investigate these effects statistically, a multiple linear regression (MLR) model was run that included a quadratic effect for the number of scale points; Table 2 shows the MLR results for the RMSEA*1000 variable.

Table 2 – MLR Results for RMSEA*1000

Coefficient	Estimate	Standard Error	t	*p*-value.
Intercept	8.238	.854	9.650	<.001
% Misresponding	.179	.198	.904	n.s.
# of Misresponding Items	.033	.144	.231	n.s.
# Scale Points	-3.345	.381	-8.775	<.001
(# Scale Points)^2	.339	.047	7.285	<.001

Note: n.s. = not significant at $\alpha = .05$.

This shows that only the number of scale points is a statistically significant predictor for the variation in RMSEA*1000 values and that the trend between the number of scale points and the RMSEA is quadratic in nature; Figure 3 depicts this trend.

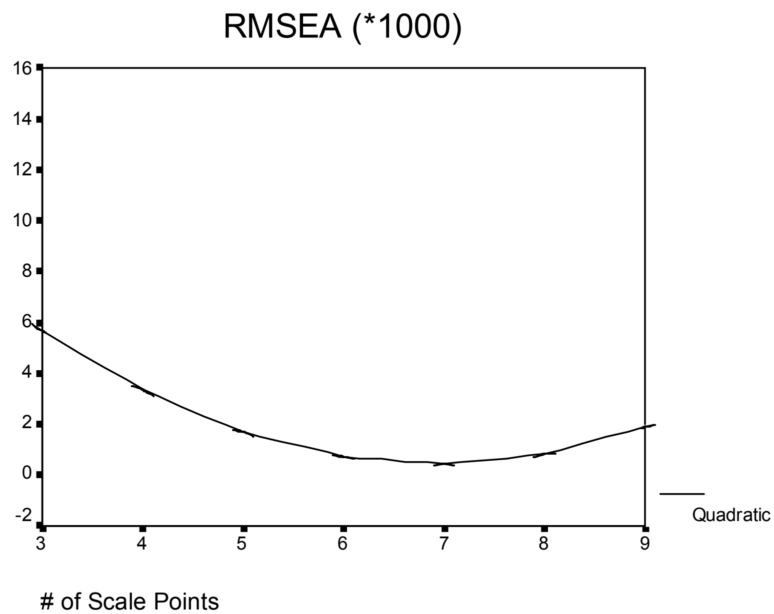


Figure 3. Quadratic trend between the number of scale points and the RMSEA values.

Conceptually, this means that the analysis picked up that there is an overall reliable difference in the RMSEA values across the different numbers of misresponding items and scale points averaging over the all remaining conditions. This model has an adjusted R^2 of .572 indicating that the quadratic trend can account for about half of the observed variation in RMSEA values. To complete the MLR perspective on the data, table 3 lists the MLR results for the REV variable.

Table 3 – MLR Results for REV

Coefficient	Estimate	Standard Error	t	*p*-value.
Intercept	5.038	.044	115.781	<.001
% Misresponding	-.004	.010	-.432	n.s.
# of Misresponding Items	.015	.007	2.104	<.05
# Scale Points	.035	.019	1.825	n.s.
(# Scale Points)^2	-.004	.002	-1.774	n.s.

Note: n.s. = not significant at $\alpha = .05$.

In contrast to the RMSEA MLR analysis, this table shows that the number of items is a statistically significant predictor for the variation in REV values. Despite the fact that the *p*-value for the quadratic effect of the number of scale points was not significant at $\alpha = .05$, it only exceeded it by a small amount giving some minor evidence for the potential of a small quadratic trend; Figure 4 shows this trend.

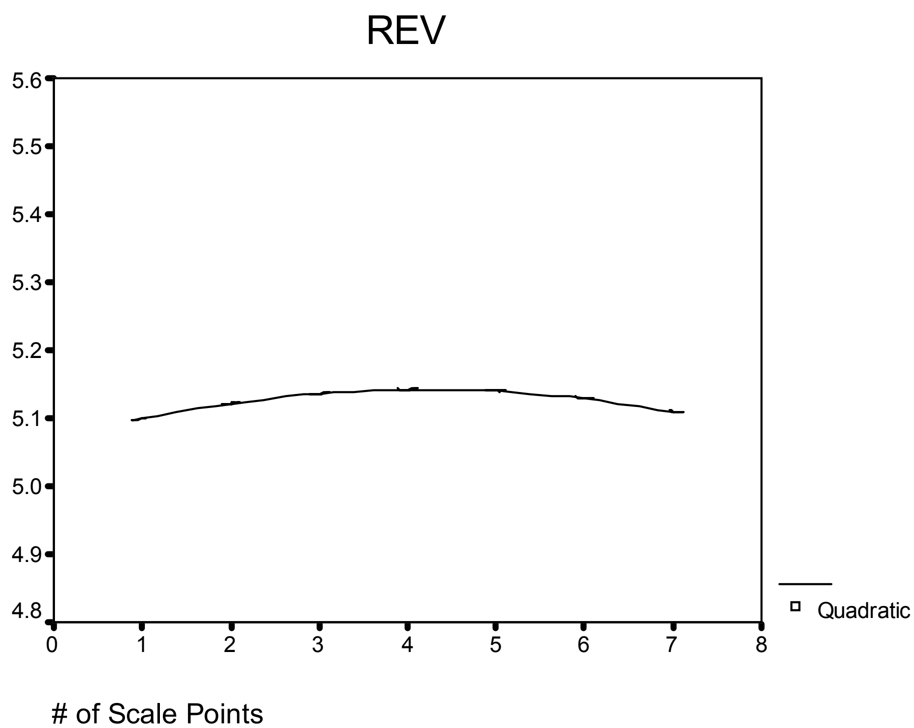


Figure 4. Quadratic trend between the number of scale points and the REV values.

One should note that the adjusted R^2 for this model is .045, though, indicating that there is little of the variation in REV can be meaningfully accounted for by any of these variables. It is also not surprising that there is only a minor quadratic trend, because the analyses based on the polychoric correlation matrix treat the data in their "proper" ordinal format and deviations from the continuous case should be less severe. Finally, it is interesting to note that for both the RMSEA and REV, the percent of examinees misresponding did not have a statistically significant effect on the variation in these variables.

Conclusion

This study investigated how a particular kind of misresponding, namely not being able to differentiate between the upper two scale points of a given LIKERT or rating scale, affects the values of two descriptive statistics, RMSEA and REV, for a one-factor model in an exploratory factor analysis under both PRELIS/LISREL and EQS frameworks. It was found that the polychoric correlation matrices produced by PRELIS/LISREL and EQS provide estimates of the correlations that can be considered identical for all practical purposes. Further analyses with PRELIS/LISREL showed that the strongest impact of misresponding can be observed for the smallest number of scale points (i.e., 3 and 4) across misresponding items. In addition, for about 5 scale points any differences in the two model statistics are negligible across simulation conditions with the statistics themselves only showing minor fluctuations around the value that they are taking under the continuous and no misresponding cases. There is some evidence for a quadratic trend between the number of scale points and the response variables, but the trend is not very pronounced as is expected. The fact that the trend is more pronounced for the RMSEA variable highlights an important secondary issue, namely that the choice of dependent variable in a simulation study may impact the interpretation of the importance of individual manipulated factors.

Rather than providing closure on this topic, this simulation study opens the door for more exciting research in this area. It is clear that the operationalization of “misresponding” used here is only a very specific case of all potential types of misresponding one can imagine. For example, one can consider a collapsing at multiple scale points at either or both ends of the scale as well as larger proportions examinees misresponding. In addition, the distributions of the simulated responses were symmetric and could be changed to non-symmetric and multimodal distributions.

One can of course envision scenarios where not all items have the same number of scale points and conceive of extending this research to inferential statistics and decision making for smaller samples but replications within each cell of a design. Finally, nothing was said about the underlying cognitive processes that might cause respondents to misrespond in the way that was simulated here and the linking of the simulation results to actual data collected on examinees with explanatory background information provides an exciting avenue for future research. But even at this point this study further adds to the growing and necessary body of research that finds that most violations of underlying modeling assumptions, which are always idealizations of reality, lead to negligible effects on sample statistics for 5 or more scale points.

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Appendix

Table A1 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (10% Misresponding, 1 Item, 3 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1				.001		.001			
2	.541	1	.001		.001	.001	.001			
3	.537	.461	1				.001		.001	
4	.507	.438	.431	1			.001			
5	.507	.432	.424	.404	1					
6	.458	.387	.382	.355	.363	1				
7	.453	.381	.384	.363	.353	.314	1			
8	.372	.311	.307	.297	.288	.264	.259	1		
9	.350	.297	.298	.283	.275	.252	.248	.202	1	
10	.332	.286	.278	.275	.265	.235	.229	.192	.183	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between PRELIS/LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A2 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (10% Misresponding, 1 Item, 9 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1									
2	.546	1								
3	.541	.455	1			.001				
4	.514	.439	.430	1						
5	.508	.428	.421	.404	1					
6	.457	.385	.379	.360	.361	1	.001			
7	.451	.379	.378	.363	.348	.318	1			
8	.373	.311	.310	.293	.287	.265	.260	1		
9	.352	.297	.295	.280	.275	.251	.245	.204	1	
10	.341	.292	.286	.279	.270	.242	.237	.197	.186	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A3 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (10% Misresponding, All 10 Items, 3 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1	-.001					-.001	-.001		
2	.547	1	-.001	-.001	-.001					
3	.545	.467	1				-.001			
4	.514	.445	.438	1		-.001	-.001			
5	.513	.437	.432	.412	1	-.001				
6	.465	.394	.389	.364	.370	1		-.001	-.001	
7	.459	.387	.390	.370	.359	.322	1	-.001	-.001	
8	.379	.320	.317	.305	.296	.272	.267	1	-.001	
9	.357	.303	.305	.291	.282	.260	.254	.210	1	
10	.345	.299	.291	.287	.278	.246	.241	.204	.194	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A4 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (10% Misresponding, All 10 Items, 9 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1				.001					
2	.546	1		.001						
3	.541	.455	1							
4	.514	.439	.430	1						
5	.508	.428	.421	.404	1	.001			.001	
6	.457	.385	.379	.360	.361	1				
7	.451	.379	.378	.363	.348	.317	1			.001
8	.373	.311	.310	.293	.287	.265	.260	1		
9	.352	.297	.295	.280	.275	.251	.245	.204	1	
10	.340	.292	.286	.279	.270	.242	.238	.197	.186	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A5 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (30% Misresponding, 1 Item, 3 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1				.001		.001			
2	.541	1	.001		.001	.001	.001			
3	.537	.461	1				.001		.001	
4	.507	.438	.431	1			.001			
5	.507	.432	.424	.404	1					
6	.458	.387	.382	.355	.363	1				
7	.453	.381	.384	.363	.353	.314	1			
8	.372	.311	.307	.297	.288	.264	.259	1		
9	.350	.297	.298	.283	.275	.252	.248	.202	1	
10	.323	.278	.272	.266	.259	.228	.224	.184	.179	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A6 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (30% Misresponding, 1 Item, 9 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1									
2	.546	1								
3	.541	.455	1			.001				
4	.514	.439	.430	1						
5	.508	.428	.421	.404	1					
6	.457	.385	.379	.360	.361	1	.001			
7	.451	.379	.378	.363	.348	.318	1			
8	.373	.311	.310	.293	.287	.265	.260	1		
9	.352	.297	.295	.280	.275	.251	.245	.204	1	
10	.340	.292	.286	.279	.270	.242	.237	.197	.186	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A7 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (30% Misresponding, All 10 items, 3 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1									
2	.560	1					-.002	-.002	-.001	-.002
3	.556	.477	1				-.001	-.001	-.002	-.001
4	.527	.458	.451	1			-.001	-.001	-.001	-.001
5	.520	.449	.443	.423	1	-.001	-.001	-.001	-.001	-.001
6	.476	.406	.402	.375	.381	1	-.001	-.001	-.001	-.001
7	.475	.402	.407	.386	.375	.340	1	-.001	-.001	-.001
8	.390	.331	.328	.317	.308	.287	.284	1	-.001	-.001
9	.371	.315	.318	.307	.296	.273	.272	.225	1	-.001
10	.360	.313	.307	.301	.292	.262	.259	.216	.211	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A8 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (No Misresponding, 3 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1				.001		.001			
2	.541	1	.001		.001		.001	.001		
3	.537	.461	1				.001			
4	.507	.438	.431	1						
5	.507	.432	.424	.404	1					
6	.458	.387	.382	.355	.363	1				
7	.453	.381	.384	.363	.353	.314	1			
8	.372	.311	.307	.297	.288	.264	.259	1		
9	.350	.297	.298	.283	.275	.252	.248	.202	1	
10	.337	.291	.284	.279	.270	.239	.233	.195	.186	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A9 – Polychoric Correlation Matrices in PRELIS/LISREL and EQS (No Misresponding, 9 Scale Points)

Items	1	2	3	4	5	6	7	8	9	10
1	1									
2	.546	1								
3	.541	.455	1			.001				
4	.514	.439	.430	1						
5	.508	.428	.421	.404	1					
6	.457	.385	.379	.360	.361	1	.001			
7	.451	.379	.378	.363	.348	.318	1			
8	.373	.311	.310	.293	.287	.265	.260	1		.001
9	.352	.297	.295	.280	.275	.251	.245	.204	1	
10	.341	.292	.286	.279	.270	.242	.237	.198	.186	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the polychoric correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.

Table A10 – Pearson's r Correlation Matrix (Continuous Case)

Items	1	2	3	4	5	6	7	8	9	10
1	1									
2	.546	1								
3	.542	.456	1							
4	.516	.438	.432	1						
5	.509	.429	.422	.406	1					
6	.456	.384	.379	.361	.361	1				
7	.451	.379	.378	.364	.349	.318	1			
8	.373	.312	.311	.295	.288	.265	.259	1		
9	.352	.298	.296	.282	.276	.251	.245	.203	1	
10	.343	.293	.288	.279	.271	.242	.239	.198	.187	1

Notes: The lower off-diagonal contains the PRELIS/LISREL computations of the Pearson correlations while the upper off-diagonal contains the difference Δ_j between LISREL and EQS correlations where Δ_j = LISREL estimate – EQS estimate. If the cell is empty, no difference exists up to the third decimal.